

Superposition of Waves

- If two or more traveling waves are moving through a medium, the resultant wave function at any point is the algebraic sum of the wave functions of the individual waves. (**Superposition principle**).

$$y_{\text{Total}}(x, t) = y_1(x, t) + y_2(x, t)$$

Superposition of two harmonic waves

- Individual wave functions:

$$y_1(x, t) = A \sin(kx - \omega t)$$

$$y_2(x, t) = A \sin(kx - \omega t - \Delta j)$$

- Add according to superposition principle

$$y(x, t) = \left(2A \cos \frac{\Delta j}{2} \right) \sin \left(kx - \omega t - \frac{\Delta j}{2} \right)$$

using trig identity for the sum of sines

$$\sin \mathbf{a} + \sin \mathbf{b} = 2 \cos \left(\frac{\mathbf{a} - \mathbf{b}}{2} \right) \sin \left(\frac{\mathbf{a} + \mathbf{b}}{2} \right)$$

Constructive and destructive Interference

$$y(x,t) = \left(2A \cos \frac{\Delta j}{2} \right) \sin \left(kx - \omega t - \frac{\Delta j}{2} \right)$$

- If $\Delta\phi=0$ or a multiple of 2π , then y_1, y_2 are in phase and we have constructive interference. The amplitude of the combined wave is twice the original amplitude.
- If $\Delta\phi=\pi$ (or an odd multiple of π), then y_1, y_2 are out of phase and we have destructive interference. The amplitude of the combined wave vanishes.
- Phase difference $\Delta\phi$ and path difference Δr :

$$\Delta r = |r_1 - r_2| = \frac{l}{2p} \Delta j \qquad \Delta j = \frac{2p\Delta r}{l}$$

Standing Waves

- If two waves with the same amplitude, frequency, wavelength, and phase travel in **opposite** directions, they form a **standing wave**. This wave does not travel.

$$y_1(x, t) = A \sin(kx - \omega t)$$

$$y_2(x, t) = A \sin(kx + \omega t)$$

$$y_T(x, t) = y_1(x, t) + y_2(x, t) = 2A \sin(kx) \cos(\omega t)$$

- We note that there are **nodes** with zero amplitude and **antinodes** with maximum amplitude. Adjacent nodes are separated by $\lambda/2$, adjacent antinodes are also separated $\lambda/2$. The distance between a node and the next antinode is $\lambda/4$.